Analysis of Discrete – Time Queueing Model with Negative customer Arrivals

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Abstract

In this paper we analyzed queue lengths and the busy period lengths of the M/G/1 queuing systems with negative arrivals. There are two types of arrivals namely negative arrivals and disasters. If there is more than one customer in a system then a positive customer is exit when a negative customer enters into system of queues. Also assume that a removal of a customer at the head type of negative customers which represent a kind of work – canceling signal to the positive customer in service. Consequently disasters get rid of all customers in the system. Some of the performance measure of the discrete – time queueing model M/G/1 is obtained and numerical examples are also given for negative customer arrivals and also disaster.

1. Introduction

Recently there has been a rapid increase in the literature on queuing systems with negative arrivals. When a negative customer arrives, then immediately removes an ordinary (positive) customer if more than one present. Negative arrivals have been interpreted as inhibiter and synchronization signals in neutral and high speed communication network.

During the last decade there has been an increasing interest in queuing system and networks with negative arrivals and their application. Negative customers systems were introduced by Gelenbe [6] with a view to modeling neural networks where a node represents a neuron. Since 1989, the name G – queue has been adopted for queues with negative customers in acknowledgement of Gelenbe.

The first variant is that of the traditional negative customer, suited to the modeling of killing signals in speculative parallelism. It can also be used to model cell losses caused by the arrival of a corrupted cell or one encountering a full buffer, when the preceding cells of a packet would be discarded. The second variant is a modification suitable for the modeling of
load balancing where work is transferred from overloaded queues but never work that is actually in progress.

There exists a further killing discipline: Removal of the customer from the head of the queue. Here, customers are removed from the head of the queue, i.e., the earliest arrivals go first. The discipline is suitable for modeling server breakdowns.

On the other hand, the analysis of discrete – time queuing models has received considerable attention in the scientific literature over the past years in view of its applicability in the study of many computer and communication systems in which time is slotted. Discrete – time queuing models are particularly appropriate to describe the various queuing related phenomena in digital computer and communication systems.

In this paper, we consider with negative arrivals in discrete – time and the killing strategy is Removal of customer at the head. Discrete – time queuing models are particularly appropriate to describe the various queuing related phenomena in digital computer and communication systems, including mobile and networks based on asynchronous transfer mode technology, due to the packetized nature of these transport protocols.

The remainder of the paper is organized as follows: In section 2, the queuing model is presented with notations. In section 3, the length of busy period is analyzed. The steady – state distribution for the number of customers in the system with negative arrivals are provided in section 4. The conclusion of this research is given in the final section.

2. Model description

This paper assumes that all events can only occur at exact slot boundaries. Customer arrives according to a Poisson arrival process. It is assumed that all queuing activities occur at the slot boundaries and the inter arrival times of positive customers are independent and identically distributed (iid). It is assumed that the service are exponentially Also, inter arrival times of negative customers and disasters are iid random variables Service discipline is a First –In,First-Out (FIFO) and service times are assumed to follow general distribution. Customers are served on a First-Come, First-Served (FCFS) basis, and service times are iid random variables with general distribution. Note that the arrivals, disasters and service times are mutually independent. Furthermore, it is assumed that when a negative customer arrives
to the system, the customer in service if more than one is removed is known as removal of customer at the head discipline.

Our objective is to find the stationary distribution by using Z – Transformation. Let A[n] is defined only for n ≥ 0, then the Z – transform is defined as

\[ A[Z] = Z\{A[n]\} = \sum_{n=0}^{\infty} A[Z]Z^n \]

3. Performance Measure

Let \( P_n(t) \) denote at a time \( t \) and the probability generating function is defined as

\[
P_0(t+\Delta t) = (1-\tau\Delta t)P_0(t) + P_1(t)\eta\Delta t + q_0(t)\Delta t \\
P_n(t+\Delta t) = \tau\Delta t P_{n-1}(t) + (1-(\tau + \eta)\Delta t)P_n(t) + \eta\Delta t P_{n+1}(t) + q_0(t)\Delta t \\
= \tau\Delta t P_{n-1}(t) + P_n(t) - \tau P_n(t)\Delta t + \eta\Delta t P_n(t) + \eta\Delta t P_{n+1}(t) + q_0(t)\Delta t \\
= \tau(P_{n-1}(t)\Delta t - P_n(t)\Delta t) + \eta(P_n(t)\Delta t + P_{n+1}(t)\Delta t) + q_0(t)\Delta t
\]

Now we obtain differential equation for the probabilities \( P_n(t) \)

\[
P_0(t) = -\tau P_0(t) + \eta P_1(t) \\
P_n(t) = \tau P_{n-1}(t) - (\tau + \eta)P_n(t) + \eta P_{n+1}(t), n=1,2,... \quad \quad (1)
\]

If \( t \to \infty \), then \( P_n'(t) \to 0 \) and \( P_n(t) \to P_n \) from equation (1), then it follows the limiting probabilities of \( P_n \),

\[
0 = -\tau P_0 + \eta P_1 \quad \quad (2) \\
0 = \tau P_{n-1} - (\tau + \eta)P_n + \eta P_{n+1}, n=1,2,... \quad \quad (3)
\]

Then the recurrence relation for equation (3) can be obtained as follows

\[
P_n = P_1Z_1^n + P_2Z_2^n, n=1,2,... \quad \quad (4)
\]

Where w= waiting time, B= random variable and S = W+B, Then \( \rho = \frac{P}{S} \) is the load of the system, then the equation can be obtained as follows
The number of customers in the system is given by \( P_L(z) = \sum_{n=0}^{\infty} A_n z^n \), by multiplying \( n^{th} \) equation with \( z^n \) and then summing the equation overall \( n \), then \( P_L(Z) \),

\[
0 = \eta P_0 (1 - Z^{-1}) + (\tau z + \eta z^{-1} - (\tau + \eta)P_L(Z)
\]

\[
P_L(z) = \sum_{n=0}^{\infty} (1 - \rho) \rho^n z^n
\]

The expected value and the minimum number of customer in the system are given by

\[
E(L) = \frac{\rho^n}{(1-\rho)^n}
\]

The mean and the minimum number of customer in the system are given by

\[
\delta_n = \sum_{n=1}^{k} (-1)^n \binom{k}{n} \frac{\rho^n}{1-\rho^n}
\]

The customer has to wait for its own service time is given by

\[
E(S) = \frac{E(L)}{\lambda}
\]

The mean waiting time \( E(W) \) can be obtained by subtracting the mean service time

\[
E(W) = \frac{\rho/\mu}{1-\rho}
\]

The mean and the minimum queue length are given by

\[
\delta_n = \frac{\rho^{2n}}{1-\rho^n}
\]

We compute the expected value of the maximum number of customers in the system and queue during each interval \( n \) is given by

\[
\rho = \Delta \delta_n - \Delta \delta_n
\]
4. Analysis of Service Time

The actual service time of ordinary customer is defined as the time period that the customer stays in server and is denoted by $S$. It may be equal service time of the customer required and may be more than the service time for the negative customer arrival. The probability generating function of $S (Z)$ is defined as follows:

$$S (Z) = \sum_{k=0}^{\infty} S(Z) z^k$$

Define $S_{ij} = \lim_{n \to \infty} P\{s_i=n/s_j = k\}$; Then it can be determined that if $n < k$, then $s_{ij} = pq^n$, and if $n=k$, then $s_{ij} = q^k$, if $n > k$, then $s_{ij} = 0$.

$$S (Z) = \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} S_{ij} s_k z^n$$

$$= \sum_{k=0}^{\infty} \sum_{n=0}^{k-1} pq^n s_k z^n + \sum_{k=0}^{\infty} q^k s_k z^k$$

$$= \frac{pqS(qz)}{1-qz} + S(qz).$$

5. Busy Period

In this study there are three assumptions can be considered for the busy period analysis namely service completion, arrival of negative customer and occurrence of a disaster. The customer can leave the system either service completion or occurrence of a disaster. The arrival of negative customer leads to two causes either customer leaves the system or the system becomes empty. $B(t)$ denote the modified busy period and its $z$-transformation is given by

$$B(t) = \{P(w > s+t /w>s)\}z^n$$

$$= \{P(w>s+t)/P(w>s)\}z^n$$

$$= (e^{-\lambda(s+t)}/e^{-\lambda s})z^n$$

$$B(t) = e^{-\lambda t} z^n \quad \text{-------- (5)}$$

We can find the expected length of the busy period by using the equation (5)

$$E[B(T)] = -\lambda e^{-\lambda t} z^n$$
The busy period is then an inter arrival time of a disaster, we can use the \( z \) – transformation to the busy period of our model \( B(t) \) we obtained as follows:

\[
D(t) = \{P(D > s + t | D > S)\} \tau^n
\]

\[
= \left(e^{-\theta t(s+t)} \big/ e^{-\theta s}\right) \tau^n
\]

\[
D(t) = e^{-\theta t} \tau^n
\]

7. Numerical Results

**Table: 1**

Table 1 shows the expected values of the maximum number of customers in the system

<table>
<thead>
<tr>
<th>n</th>
<th>( \delta_n )</th>
<th>( \delta_n' )</th>
<th>( \rho )</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>0.365654</td>
<td>0.365654</td>
<td>1.07789E-11</td>
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<td>2</td>
<td>0.421909</td>
<td>0.421909</td>
<td>3.8287E-10</td>
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<td>3</td>
<td>0.49862</td>
<td>0.49862</td>
<td>1.37818E-8</td>
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<td>4</td>
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<td>0.609423</td>
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<td>0.685602</td>
<td>0.685599</td>
<td>2.97687E-6</td>
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<td>6</td>
<td>0.783545</td>
<td>0.783527</td>
<td>0.00001786</td>
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<tr>
<td>7</td>
<td>0.914135</td>
<td>0.914028</td>
<td>0.0001071</td>
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<tr>
<td>8</td>
<td>1.09697</td>
<td>1.09632</td>
<td>0.000643</td>
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<td>9</td>
<td>1.37125</td>
<td>1.36739</td>
<td>0.003858</td>
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<td>10</td>
<td>1.82817</td>
<td>1.80502</td>
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**Table: 2**

Table 2 shows minimum number of customer in the system

<table>
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<td>5</td>
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<td>6</td>
<td>0.503529</td>
<td>0.16863</td>
</tr>
<tr>
<td>7</td>
<td>0.38712</td>
<td>0.108038</td>
</tr>
</tbody>
</table>
6. Conclusion

We have successfully analyzed the queuing systems with negative arrivals, performance measure of the queue length, Analysis of service time and busy period. The negative arrival leads to damage the workload of the system. Though we obtain the probability generating function for performance measure and also we derive the expected value for busy period we obtain the probability generating function of the performance measure of busy period by recursion analysis. It is shown here that the results can be easily be dealt with numerically.

7. Reference