International journal of basic and applied research
www.pragatipublication.com
ISSN 2249-3352 (P) 2278-0505 (E)
Cosmos Impact Factor-5.86

An application of assignment problem "airlines crew assignment problem and its solution"

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Abstract: Assignment problems arise in different situation where we have to find an optimal way to assign n objects to m-other objects in an injective fashion. The assignment problem is a special case of Transportation problem. Depending on the objective we want to optimize, we obtain the typical assignment problems. Assignment problem is an important subject discussed in real physical world. Here in this paper, we discuss an application of Assignment problem that can be seen in Airlines services and comparison of its solution by well known method i.e Hungarian method and new modified method DAM method. This application is named as Airlines Crew Assignment Problem. In this assignment problem we have to plan the assignment of crew members at different locations by a transport company.

Keywords: Assignment problems, Airlines crew assignment problems, Hungarian assignment (HA) method, modified approach method (Division Algorithm Method), optimization.

To enhance the understanding about this assignment model, we provide the following example.

Example: Airlines Crew Assignment Problem: Best-ride airlines that operates seven days a week has the following time-table.

Flight No.	Departure	Arrival
101	07:00 AM	08:00 AM
102	08:00 AM	09:00 AM
103	01:00 PM	02:00 PM
104	06:00 PM	07:00 PM

Received: 5 March Revised: 13 March Accepted: 22 March
Index in Cosmos
April 2018 Volume 8 Number 4
UGC APPROVED

International journal of basic and applied research
www.pragatipublication.com
ISSN 2249-3352 (P) 2278-0505 (E)
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Table-(A) Delhi – Lucknow

Flight No.	Departure	Arrival
101	08:00 AM	09:00 AM
102	09:00 AM	10:00 AM
103	12:00 Noon	01:00 PM
104	05:00 PM	06:00 PM

Table-(B) Lucknow – Delhi.

Crews must have a minimum layover of 5 hours between flights. Obtain the pairing of flights that minimizes layover time away from home. For any given pairing, the crew will be based at the city that results in the smaller layover. For each pair also mention the city where crew should be based.

Solution:

To determine optimal assignments, first we calculate layover times from the above time table.Calculating values for table 1 (layover time)

First Row

First cell:
Arrival time (Lucknow) = 8.00 AM & Departure time (Lucknow) = 8.00 AM
Difference between arrival and departure = 24 hours (layover time)
Second cell:
Arrival time (Lucknow) = 8.00 AM & Departure time (Lucknow) = 9.00 AM
Difference between arrival and departure = 25 hours (layover time)
Third cell:
Arrival time (Lucknow) = 8.00 AM & Departure time (Lucknow) = 12.00 Noon
Difference between arrival and departure = 28 hours (layover time)
Fourth cell:
Arrival time (Lucknow) = 8.00 AM & Departure time (Lucknow) = 5.00 PM
Difference between arrival and departure = 9 hours (layover time).
Similarly, values for other rows can be calculated.

Received: 5 March Revised: 13 March Accepted: 22 March
Index in Cosmos
April 2018 Volume 8 Number 4
UGC APPROVED

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www.pragatipublication.com
ISSN 2249-3352 (P) 2278-0505 (E)
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Flight No.	101	102	103	104
01	24	25	28	9
02	23	24	27	8
03	18	19	22	27
04	13	14	17	22

` **Table.1 Layover time matrix for Crew based at Delhi**

Now calculating values for table 2 (layover time)

First Column

First cell:

Arrival time (Delhi) = 9.00 AM & Departure time (Delhi) = 7.00 AM

Difference = 22 hours

Second cell:

Arrival time (Delhi) = 9.00 AM & Departure time (Delhi) = 8.00 AM

Difference = 23 hours

Third cell:

Arrival time (Delhi) = 9.00 AM & Departure time (Delhi) = 1.00 PM

Difference = 28 hours

Fourth cell:

Arrival time (Delhi) = 9.00 AM & Departure time (Delhi) = 6.00 PM

Difference = 9 hours

Similarly, values for other columns can be calculated.

Flight No.	101	102	103	104
01	22	21	18	13
02	23	22	19	14
03	28	27	24	19
04	9	8	5	24

Table.2 Layover time matrix for Crew based at Lucknow.

Further, the composite layover time matrix (table 3) is obtained by selecting the smaller element from the two corresponding elements of table 1 & 2. The layover time marked with **(*)** represents

Received: 5 March Revised: 13 March Accepted: 22 March
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UGC APPROVED

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ISSN 2249-3352 (P) 2278-0505 (E)
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that the crew is based at Lucknow, otherwise based at Delhi. For example, corresponding to flight no.1 and 101 in table 1 & 2, we select the minimum between (24, 22), i.e., 22 for Lucknow. Therefore, this element is marked with **(*)**. In this way, table 3 is completed and shown below.

Flight No.	**101**	**102**	**103**	**104**
01	22*	21*	18*	9
02	23	22*	19*	8
03	18	19	22	19*
04	9*	8*	5*	22

Table.3 Composite layover time matrix

Now the above problem can be easily solved by Hungarian method and DAM method.

Solution by Hungarian method:

Step-1 Choosing the minimum most elements in each row and subtracting the same we obtain the matrix as follows-

Flight No.	**101**	**102**	**103**	**104**
01	13	12	9	0
02	15	14	11	0
03	0	1	4	1
04	4	3	0	17

Step-2 Choosing the minimum most elements in each column and subtracting the same we obtain the matrix as follows-

Flight No.	**101**	**102**	**103**	**104**
01	13	11	9	0
02	15	13	11	0
03	0	0	4	1

Received: 5 March Revised: 13 March Accepted: 22 March
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UGC APPROVED

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www.pragatipublication.com
ISSN 2249-3352 (P) 2278-0505 (E)
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04	4	2	0	17

Step-3 Assigning all the zeroes by their priority and covering them with minimum number of lines we obtain the matrix as follows-

Flight No.	**101**	**102**	**103**	**104**
01	13	12	9	0 □
02	15	14	11	0 ⊗
03	0 □	1	4	1
04	4	2	0 □	17

Step-4 Since number of rows drawn is not equal to the order of the matrix hence the assignment is not optimal.

To get the optimal assignment we choose the minimum most elements from uncovered area. Subtracting this element from all uncovered elements and adding it to the junction of the lines we have the following matrix-

Flight No.	**101**	**102**	**103**	**104**
01	4	3	0	0
02	6	4	2	0
03	0	1	4	10
04	4	2	0	26

Step-5 Again repeating step 3 we get the matrix as follows-

Flight No.	**101**	**102**	**103**	**104**
01	4	3	0 □	0 ⊗
02	6	4	2	0 □
03	0 □	1	4	10
04	4	2	0 ⊗	26

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UGC APPROVED

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Step-6 Since number of rows drawn is not equal to the order of the matrix hence the assignment is not optimal again.

To get the optimal assignment we choose the minimum most elements from uncovered area. Again subtracting this element from all uncovered elements and adding it to the junction of the lines we have the following matrix-

Flight No.	**101**	**102**	**103**	**104**
01	2	0	0	0
02	4	2	2	0
03	0	0	6	12
04	2	0	0	26

Step-7 Again repeating step 3 we get the matrix as follows-

Flight No.	**101**	**102**	**103**	**104**
01	2	0 □	0 ⊗	0 ⊗
02	4	2	2	0 □
03	0 □	0	6	12
04	2	0	0 □	26

Obviously, the number of rows drawn is equal to the order of the matrix. Hence the assignment is optimal.

Thus, the optimal solution is 21 + 8 + 18 + 5 = 52 Hours.

Received: 5 March Revised: 13 March Accepted: 22 March
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April 2018 Volume 8 Number 4
UGC APPROVED

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Cosmos Impact Factor-5.86

Solution by modified method i.e DAM method:

Now we try to solve this Airlines Crew Assignment Problem by modified method in following steps-

Step 1- Choosing the minimum most elements in each row and write it to the right of that row.

Flight No.	101	102	103	104	Min.
01	22*	21*	18*	9	9
02	23	22*	19*	8	8
03	18	19	22	19*	18
04	9*	8*	5*	22	5

Step 2- Dividing each row by the minimum most elements in that row we get a new matrix as-

Flight No.	101	102	103	104	Min.
01	22/9	21/9	18/9	1	9
02	23/8	22/8	19/8	1	8
03	1	19/18	22/18	19/18	18
04	9/5	8/5	1	22/5	5

Step 3- Choosing the minimum most elements in each column and write it to the downside of that column.

Flight No.	101	102	103	104	Min.
01	22/9	21/9	2	1	9
02	23/8	22/8	19/8	1	8
03	1	19/18	22/18	19/18	18
04	9/5	8/5	1	22/5	5
Min.	1	19/18	1	1	

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Step 4- Dividing each column by the minimum most elements in that column we get a new matrix as-

Flight No.	101	102	103	104	Min.
01	22/9	42/19	2	1	9
02	23/8	99/38	19/8	1	8
03	1	1	22/18	19/18	18
04	9/5	144/95	1	22/5	5
Min.	1	19/18	1	1	

Step 5- Assigning all the ones by their priority order and covering them by minimum number of lines we have-

Flight No.	101	102	103	104	Min.
01	22/9	42/19	2	1 □	9
02	23/8	99/38	19/8	1 ⊗	8
03	1 □	1 ⊗	22/18	19/18	18
04	9/5	144/95	1 □	22/5	5
Min.	1	19/18	1	1	

Step 6- Since the number of lines are not equal to the order of matrix. Therefore the assignment is not optimal in such case we choose the minimum most element from the uncovered region and dividing the uncovered elements by the same. We have the new matrix as follows-

Flight No.	101	102	103	104	Min.
01	11/9	21/19	1	1	9
02	23/16	99/76	19/16	1	8
03	1	1	22/18	19/18	18
04	9/5	144/95	1	22/5	5
Min.	1	19/18	1	1	

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Step 7- Further again assigning all the ones by their priority order and covering them by minimum number of lines we have-

Flight No.	101	102	103	104	Min.
01	11/9	21/19	1 □	1 ⊗	9
02	23/16	99/76	19/16	1 □	8
03	1 □	1 ⊗	22/18	19/18	18
04	9/5	144/95	1 ⊗	22/5	5
Min.	1	19/18	1	1	

Step 8- Again since the number of lines are not equal to the order of matrix. Therefore the assignment is not optimal in such case repeating the step 6 and 7 we have a new matrix as-

Flight No.	101	102	103	104	Min.
01	209/89	1 □	1 ⊗	1 ⊗	9
02	437/352	99/84	19/16	1 □	8
03	1 □	1 ⊗	22/18	19/18	18
04	57/35	48/35	1 □	22/5	5
Min.	1	19/18	1	1	

Here, we see the number of lines drawn is equal to the order of the matrix thus the we have the optimal assignment.

Thus, the optimal solution is 21 + 8 + 18 + 5 = 52 Hours.
From the above discussion we observe that both the methods Hungarian method & modified method named as Division algorithm method leads the same optimal solution.

Received: 5 March Revised: 13 March Accepted: 22 March
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April 2018 Volume 8 Number 4
UGC APPROVED

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ISSN 2249-3352 (P) 2278-0505 (E)
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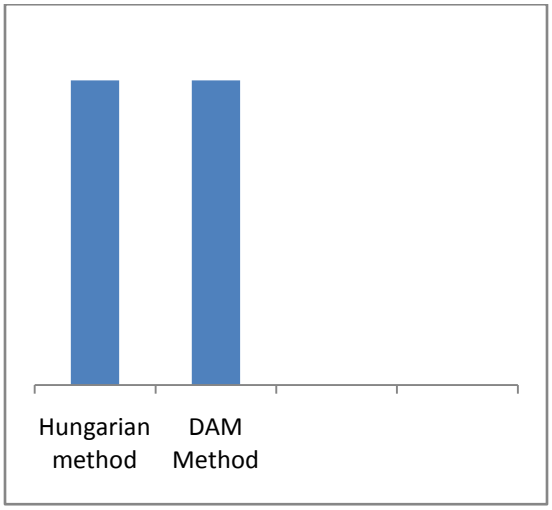
The graphical representation for the comparison of optimal cost by both the discussed methods is shown below-



Conclusion:

In this chapter, an important daily life related application of assignment problem is discussed. This application of assignment problem can be seen in Airlines services to plan the assignment of crew members at different locations by a transport company.This application is named as Airlines Crew Assignment Problem. Further, we have got the optimal solution of such an assignment problem by using well known algorithm i.e Hungarian Algorithm and modified method i.e DAM method. Both the method lead the same optimal result that is shown by graph as above.

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Received: 5 March Revised: 13 March Accepted: 22 March
Index in Cosmos
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UGC APPROVED

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Cosmos Impact Factor-5.86

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